Who am I?

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Teacher – 12 years

Content Writer

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Edrolo

Big picture changes in VIC 2.0

Year level	Change
Year 7	Circles moved to measurement (VC27M03) Manipulating formulas describing the effect of variation in the value of variables (VC2M7A06)
Year 8	3D Cartesian Coordinate system (VC2M8SP03) Pythagoras' theorem (VC2M8M06)
Year 9	Linear and quadratic equations (VC2M9A02) Algorithms design test and refine based on geometric constructions and theorems (VC2M9SP03)
Year 10/10A	Networks (VC2M10SP02) Algorithms using data structure and pseudocode (VC2M10A06) Rates of change and limiting values (VC2M10AM02) Circle theorems (VC2M10ASP01)

What is our destination?

7-10 Maths journey all leads to one destination.

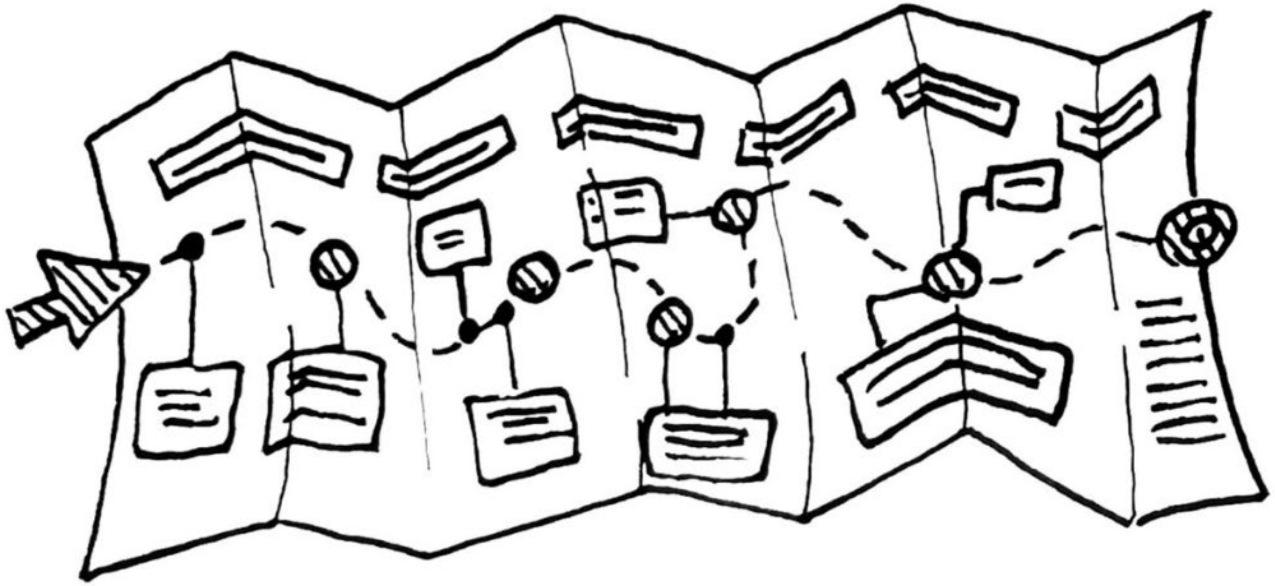
VCE General

VCE Methods

VCE Specialists

VCE Economics

VCE Accounting



Main takeaways

- Maths is maths
- New emphasis on modelling and investigation
- Algorithmic thinking

Why investigation tasks

the education hub

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Engaging and effective teaching and learning in secondary school mathematics and statistics

Home › School resources › Mathematics › Engaging and effective teaching and learning in secondary school mathematics and statistics

February 24, 2023

Mathematics and statistics contribute powerful tools for finding solutions to the world's societal, environmental, humanitarian, and economic challenges. Mathematics and statistics are essential for fighting inequity, demanding social justice, saving our environment, and developing collaboration and understanding between people. Successful secondary mathematics and statistics learning is highly empowering. It gives students options for continued study and employment. It helps them interpret, understand, and think critically about mathematical ideas and enables them to engage in society. How secondary school mathematics and statistics (subsequently collectively referred to as mathematics) is taught can make an important difference for students and what they will do in their lives with and for others. This guide presents ideas about what makes for effective and inspiring secondary school mathematics teaching and learning, with a particular focus on Aotearoa New Zealand.

Inspiring and effective secondary school mathematics teaching deftly and deliberately weaves together many areas of knowledge for teaching. Teachers need to know themselves and their learners well, their mathematics content and its history and real-life applications, how to assist students to recognise and move past partial mathematical understandings, and how to set up motivating investigative and discussion-based learning experiences that develop understanding and retention. Teachers need to convey their own joy in mathematical endeavour and nourish such joy in students. They need to know how to help students see themselves as competent,

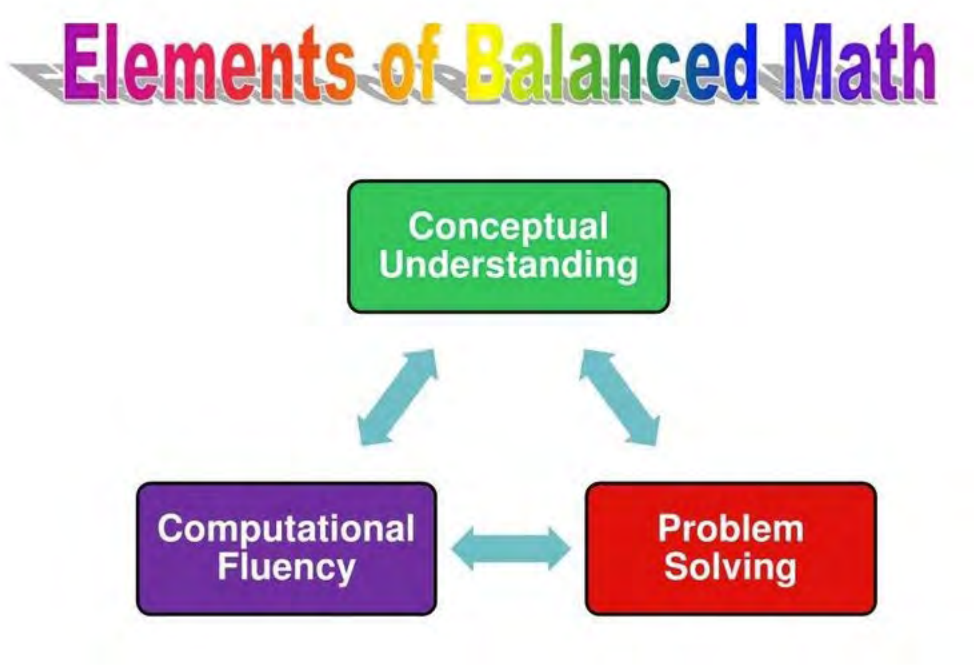
Thinking about thinking

“Mathematicians typically develop intuitive ideas before a formal proof, but we rarely ask students in K-16 mathematics education to use their intuition or to think creatively about mathematics – these important acts are devalued or completely absent.”

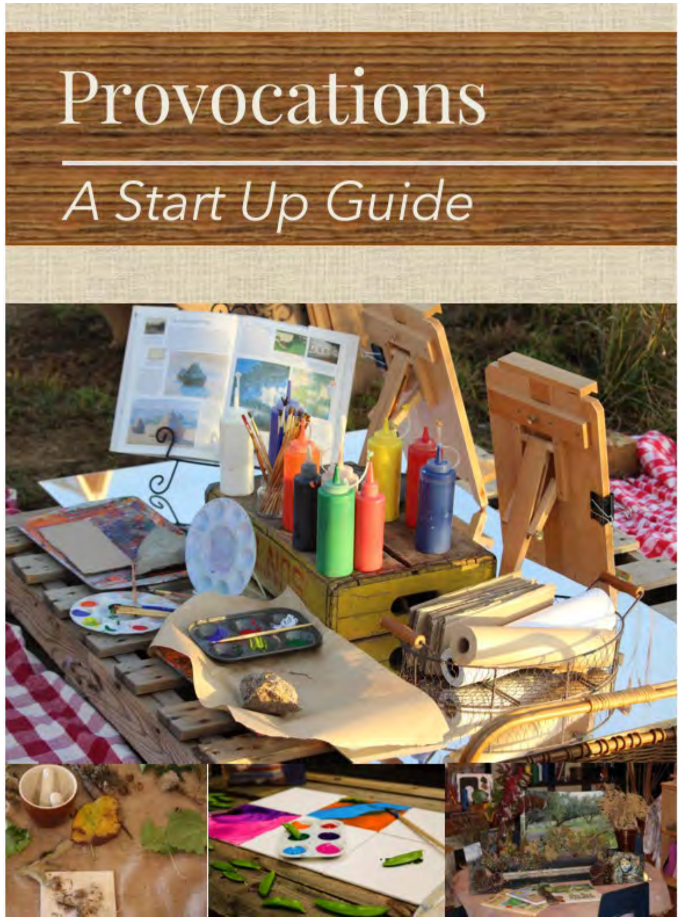
Balanced maths program

- Hands on investigation and modelling
- Investigation tasks
- Questioning
- Mathematical reasoning
- Exploration

- Explicit teaching
- Spaced repetition
- Repetitive practice



Provocations



Sally Haughey and Nicole Hill

A provocation is a verb.

It engages and activates children's thinking.

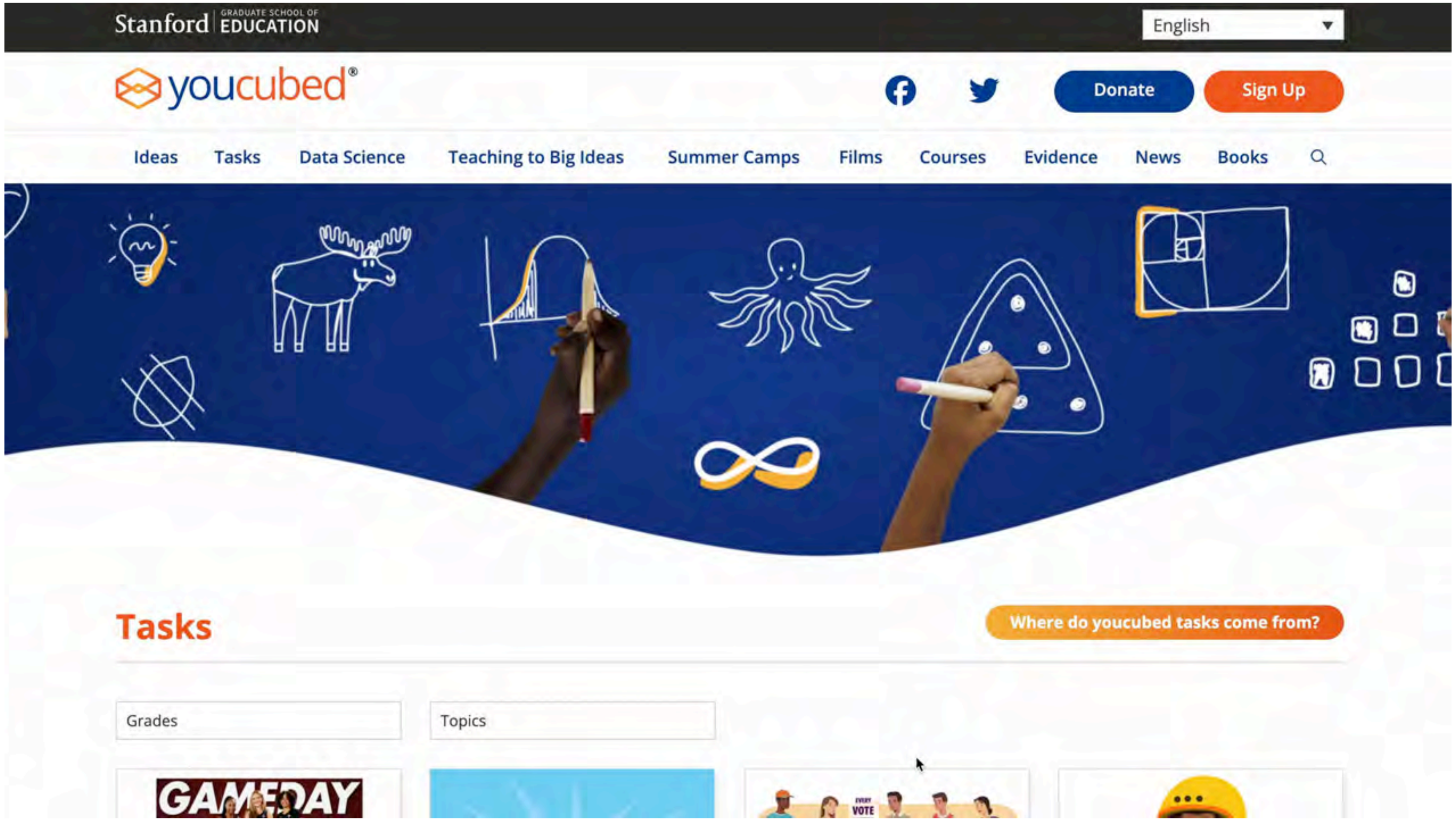
It provides children with new experiences and connections in their pursuits of ideas, interests, and theories.

A provocation ***challenges*** *the next level of thinking in the child.*

Investigation tasks

youcubed®

youcubed.org/tasks

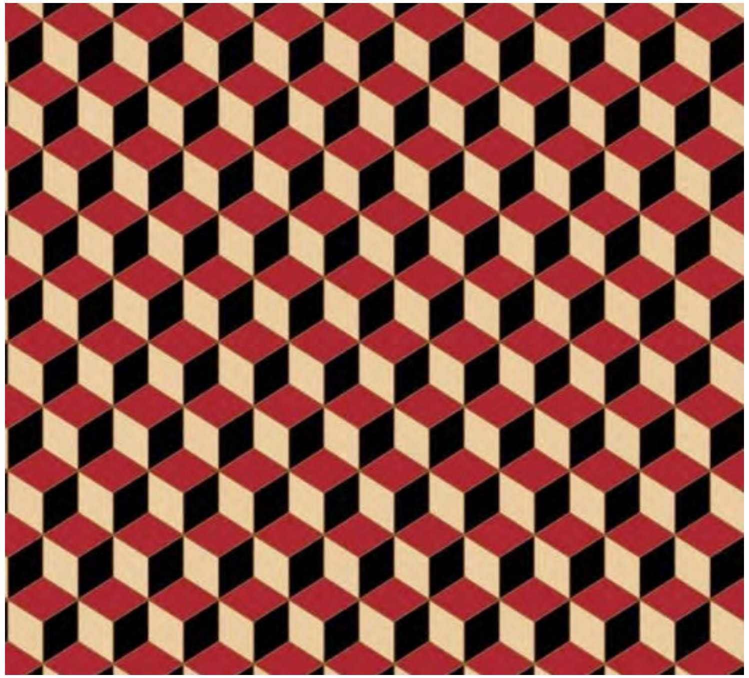


Investigation Tasks – Year 7/8

- Introduction to algebra
  - Identifying patterns
    - *generate tables of values from visually changing patterns or the rule of a function; describe and plot these relationships on the Cartesian plane (VC2M7A05)*
    - *investigate, interpret and describe relationships between variables represented in graphs of functions developed from authentic data (VC2M7A04)*
    - *manipulate formulas involving several variables using digital tools, and describe the effect of systematic variation in the values of the variables(VC2M7A06)*

Investigation Tasks – Year 7/8

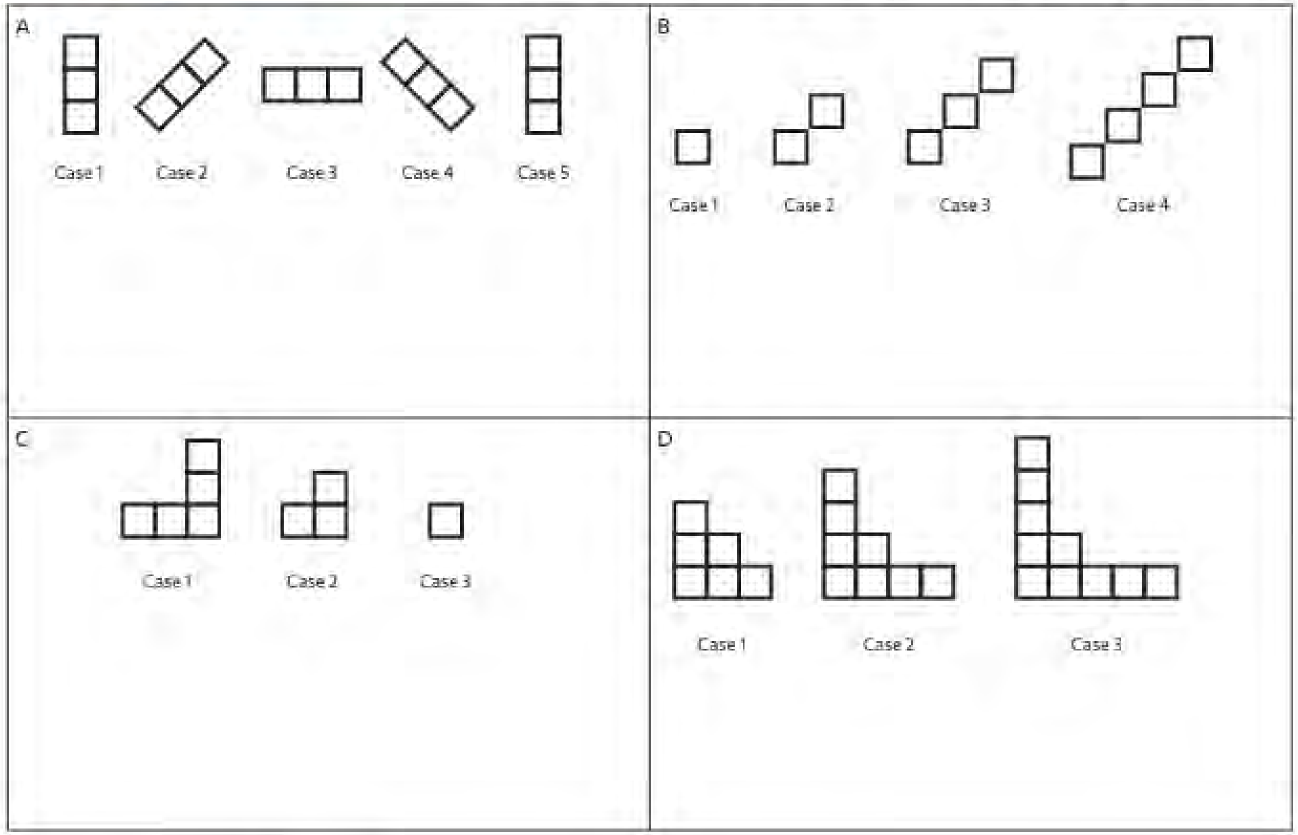
- Seeing and describing linear functions
  - youcubed (2018)



Investigation Tasks – Year 7/8

Task Cards

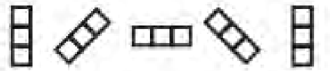
How do you see the shapes change as the case number increases? Where do you see the new squares? How do you see the shapes change as the case number decreases? What would the 15th case look like? What would the -3 case look like?



Investigation Tasks – Year 7/8

youcubed®

Algebra Representation A



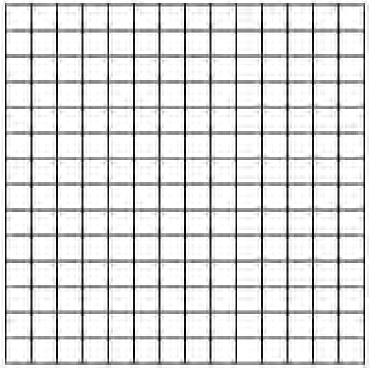
Make a table using numbers.	Make a coordinate graph to illustrate the pattern.
Describe the way the pattern is increasing or decreasing.	**Describe your function using an algebraic expression that shows the number of blocks in any case number.**

Investigation Tasks – Year 7/8

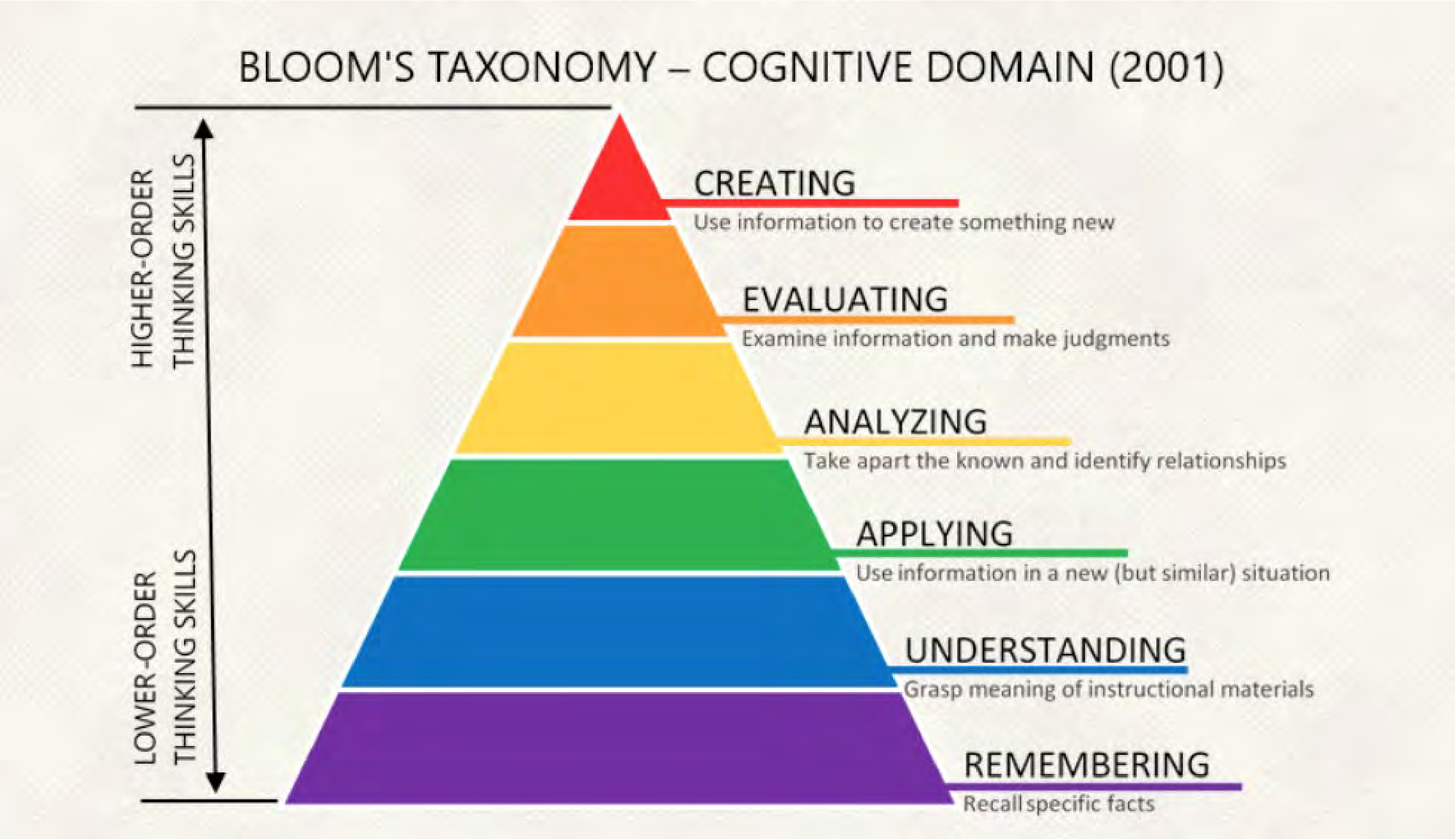
youcubed

Make your own #1

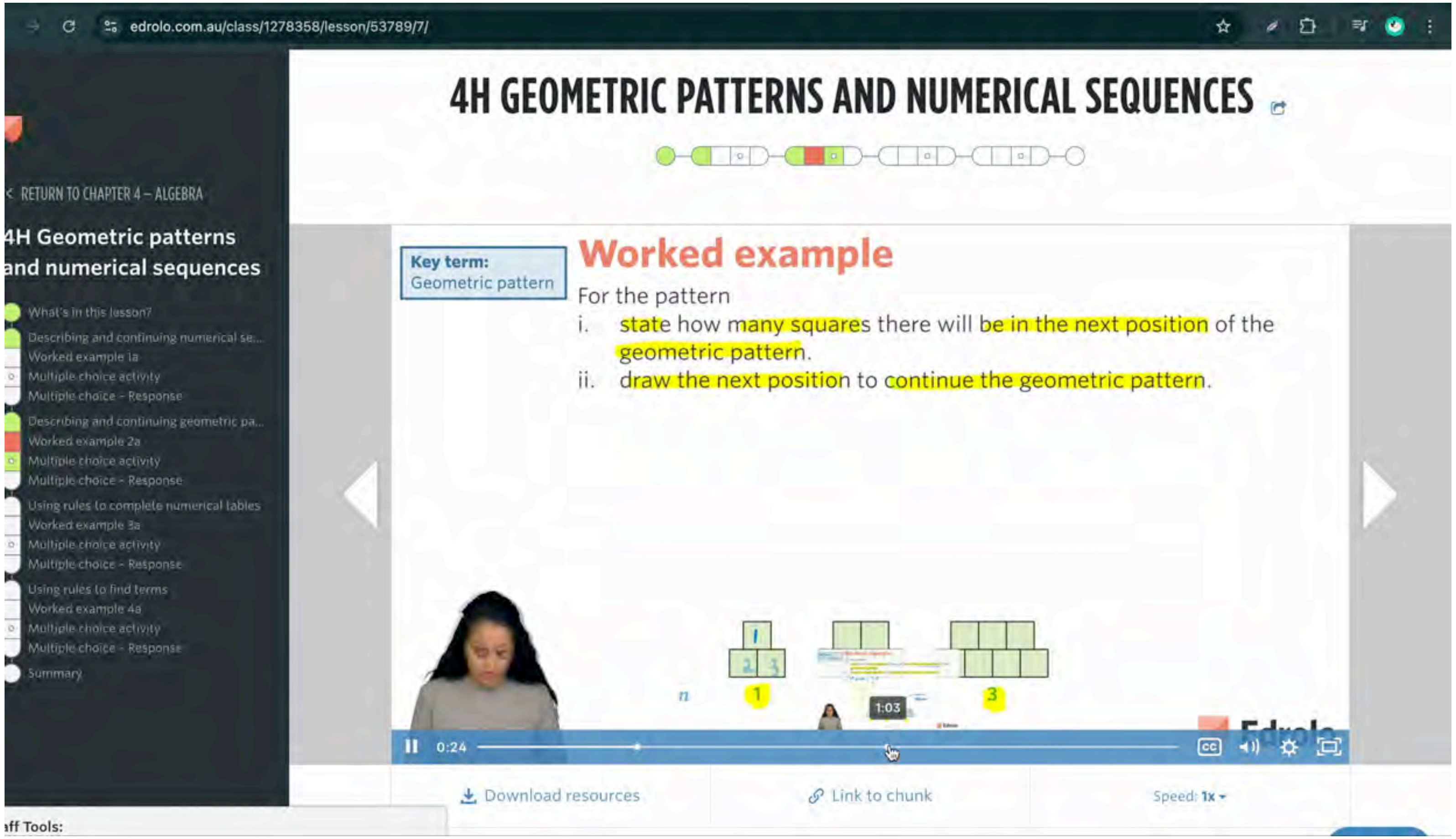
Make a pattern that grows as the case numbers increase.

Draw your pattern. Include at least 3 representations and label them by case number.	Make a table using numbers.
Make a coordinate graph to illustrate the pattern. 	Describe your function using an algebraic expression that shows the number of blocks in any case number.

Investigation Tasks – Year 7/8



Explicit teaching

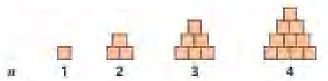


Explicit teaching

Key ideas

1. The pronumeral n refers to the **position** of each term in a geometric pattern or numerical sequence.

   In the pattern shown, $n = 1$ describes the first position in the pattern, $n = 2$ the second position, $n = 3$ the third position and $n = 4$ the fourth position.



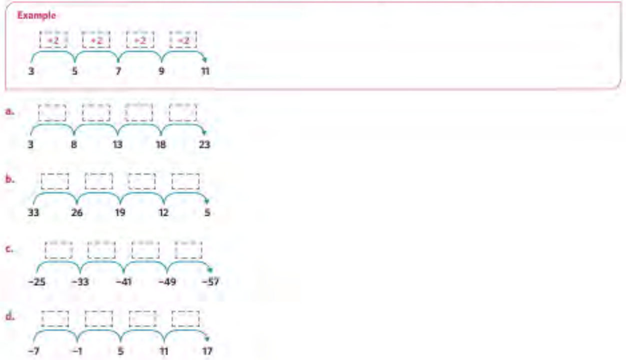
2. Tables can be used to represent numerical sequences and number patterns.

Position (n)	1	2	3	4
Number of squares (s)	2	4	6	8

3. A numerical sequence is where the values increase or decrease in a consistent way.

Continues →

2. Identify the common difference between consecutive terms and then complete the numerical sequence.



Worked example 2

Describing and continuing geometric patterns

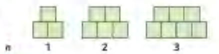
For each pattern:

i. State how many squares there will be in the next position of the geometric pattern.

ii. Draw the next position to continue the geometric pattern.

WE2a

a.



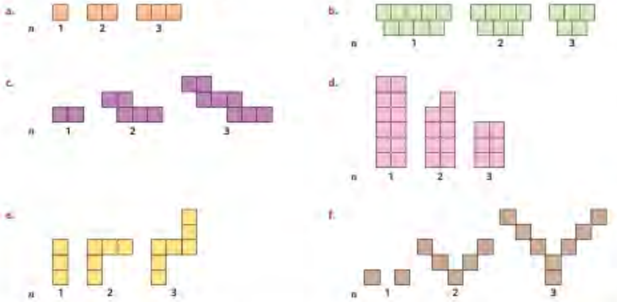
Working	Thinking
i. 3, 5, 7 The common difference is +2.	i. Step 1: Count the number of squares in the first, second and third position and write them as a numerical sequence. Find the common difference between consecutive terms.
$7 + 2 = 9$ There will be 9 squares in the next position of the geometric pattern.	Step 2: Apply the common difference to the last given term in the sequence to find out how many squares there will be in the next position.
ii. 	ii. Step 1: Identify the difference of squares between each position. Each following position adds two squares, one to the top row and one to the bottom.
	Step 2: Continue the geometric pattern by adding two squares in the appropriate locations to the pattern.

WE2

5. For each pattern:

   i. State how many squares there will be in the next position of the geometric pattern.

   ii. Draw the next position to continue the geometric pattern.



WE3

6. Use the rule to complete each table of values.

a. $t = 5n - 3$

Position (n)	1	2	3	4	5
Term (t)					

b. $t = 7n + 3$

Position (n)	1	2	3	4	5
Term (t)					

c. $t = 2n + 10$

Position (n)	3	4	5	6	7
Term (t)					

d. $t = 4n - 20$

Position (n)	8	9	10	11	12
Term (t)					

e. $t = 50n - 150$

Position (n)	9	7	5	3	1
Term (t)					

Position (n)	10	12	14	16	18
Term (t)					

f. $t = \frac{n}{4} + 6$

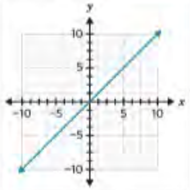
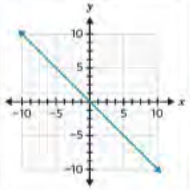
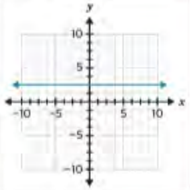
Investigation Tasks – Year 9/10

- Thinking about calculus – curved shapes
  - Discover
  - Present
  - Pair and compare
  - Whole group

Explicit teaching

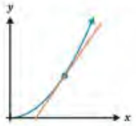
Key ideas

1. The gradient of a relation can be analysed to describe how the dependent variable is changing with respect to the independent variable.

Behaviour of the relation	y increases as x increases	y decreases as x increases	y remains constant as x increases
Gradient	Positive	Negative	Zero
Graph			

2. The instantaneous rate of change of a relation at any point is equivalent to the gradient of the graph at that point.

For a linear relation, the instantaneous rate of change at any point can be found from the gradient of the relation. For a non-linear relation, since the instantaneous rate of change is found at one specific point, the gradient cannot be determined exactly at all points without the study of calculus.



Instantaneous rate of change = gradient

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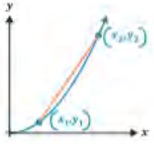
3. The average rate of change of a graph across an interval can be found by determining the gradient of a straight line connecting the two points on the relation.



Average rate of change $= \frac{y_2 - y_1}{x_2 - x_1}$

Worked example 3

Calculating average rates of change

Calculate the average rate of change of the following relations, across the specified interval.

a. $y = 3x^2 + 2$, between $x = 0$ and $x = 2$.

WE3a

Working

When $x = 0$:

$y = 3(0)^2 + 2$
$= 2$

When $x = 2$:

$y = 3(2)^2 + 2$
$= 12 + 2$
$= 14$

$\therefore$ (0,2) and (2,14)

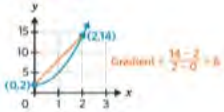
Thinking

Step 1: Identify the two points on the relation that can be used to find the average rate of change.

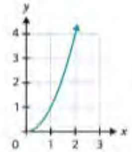
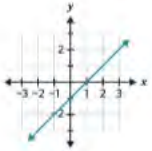
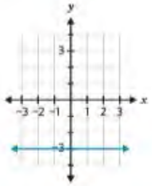
$$\text{Average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 2}{2 - 0} = \frac{12}{2} = 6$$

Step 2: Substitute the coordinates into the formula to find the average rate of change.

Visual support



2. Match the following relations to their behaviour.

Relation	Behaviour
	Decreasing at a constant rate.
	Not changing.
	Increasing at a constant rate.
	Increasing with a rate that is increasing in value.

Worded problems

Problem solving

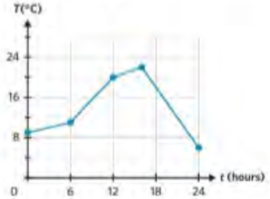
Question working paths

Mild	Medium	Spicy
10, 11, 12	11, 12, 13	12, 13, 14

10. A water tank is filled at a constant rate of 5 litres per minute for 10 minutes. Draw a graph to represent the volume of water in the tank over time. Assume the water tank is initially empty.

11. A straight road on a mountain is modelled by the relation $y = \frac{1}{2}x + 100$, where y represents the height of the road in metres, and x is the horizontal distance in kilometres from the base of the mountain. Describe the rate of change of the road's height with respect to horizontal distance from the base of the mountain.

12. The graph of the temperature, T(°C), each hour from the start of the day, t, of Melbourne during a spring day is shown.



During which part of the day is the temperature of Melbourne increasing at the greatest rate?

Reasoning

Question working paths

Mild	Medium	Spicy
15 (a,b,d)	15 (a,b,d), 16 (a,b)	All

15. Lual has a list of $(n + 1)$ numbers and needs to determine their average. Four of the numbers are $5, n^3, n^4$ and n. The remaining $(n - 3)$ numbers are all equal to 1.

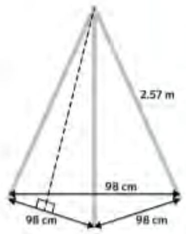
- **a.** Calculate the average of the list of numbers.
- **b.** Lual realises that n^4 and n are outliers so he removes these numbers. Calculate the new average.
- **c.** Lual has another list of $2n$ numbers, all of which have a value of 2. He combines this list with the list from part **b**. Calculate the average of the two combined lists.
- **d.** Why is it often important to remove outliers when calculating averages?

16. Let $P(x) = -x^3 + 3mx^2 + m^2x - m^3$.

- **a.** Determine the value of $P(m)$.
- **b.** Divide $P(x)$ by $(x - m)$ and identify the remainder.
- **c.** What do the answers from parts **a** and **b** show about the remainder when a polynomial is divided by a linear polynomial of the form $x - a$?

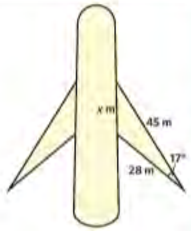
Chapter 3 extended application

1. A photographer is setting up their tripod for a photoshoot. The tripod is constructed as shown and consists of four isosceles triangles.



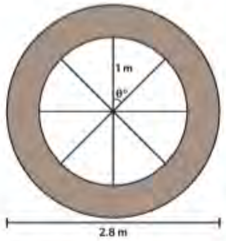
- **a.** Calculate the height of one of the isosceles triangles made by two of the tripod's legs, in metres, correct to two decimal places.
- **b.** Determine the size of each angle within the isosceles triangles to the nearest degree.
- **c.** If the distance from the bottom of one leg to the centre of the tripod is 57 cm, what angle does the tripod's leg make with the ground to the nearest degree?
- **d.** A model stands 5.6 m away from the tripod and is 179 cm tall. If a camera adds 14 cm to the height of the tripod, determine the angle of depression so that the camera captures the model's full body. Give your answer correct to one decimal place.
- **e.** Identify a reason why a photographer might use a tripod.

2. A group of engineers has designed a new aircraft for international travel that aims to reduce flight times between countries. The aircraft has a longer body than the company's regular planes and a narrower wingspan as shown.



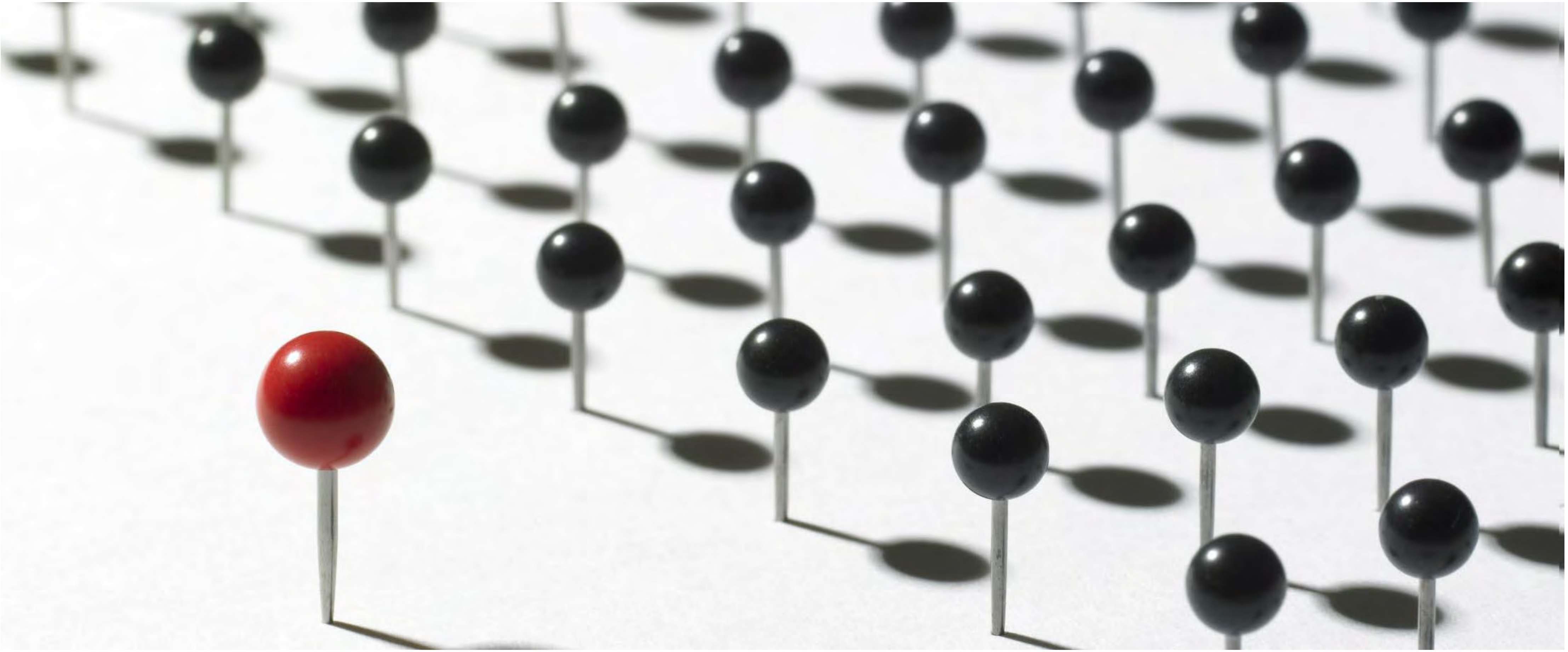
- **a.** Determine the value of x, correct to two decimal places.
- **b.** The company's regular aircraft have triangular wings with angles of 90°, 62°, and 28°. Calculate the difference, correct to two decimal places, between the largest angle of the company's regular wings to that of the new design.
- **c.** The plane takes off from the end of a runway at an angle of elevation of 15° and continues ascending at this angle until it is 12 km above the ground. At the end of its ascent, what is the horizontal distance of the plane from the point of takeoff, to the nearest kilometre?
- **d.** During testing, the plane initially flies 7648 km on a bearing of $N\ 19°\ W$ before turning and travelling 2902 km on a bearing of 262° T. How far west is the plane from its initial take-off point, to the nearest kilometre?
- **e.** The test plane's flight path can be modelled by $h = 12\sin(15t), 0 \le t \le 12$ where h is the vertical height of the aircraft in kilometres at time t hours after take-off. Sketch the graph of the test plane's flight path.
- **f.** Identify an advantage of quicker international travel times.

3. A water wheel is being remodelled to efficiently produce hydro power. The wheel has a diameter of 2.8 m and consists of eight equally spaced beams that are each 1 m in length as shown.

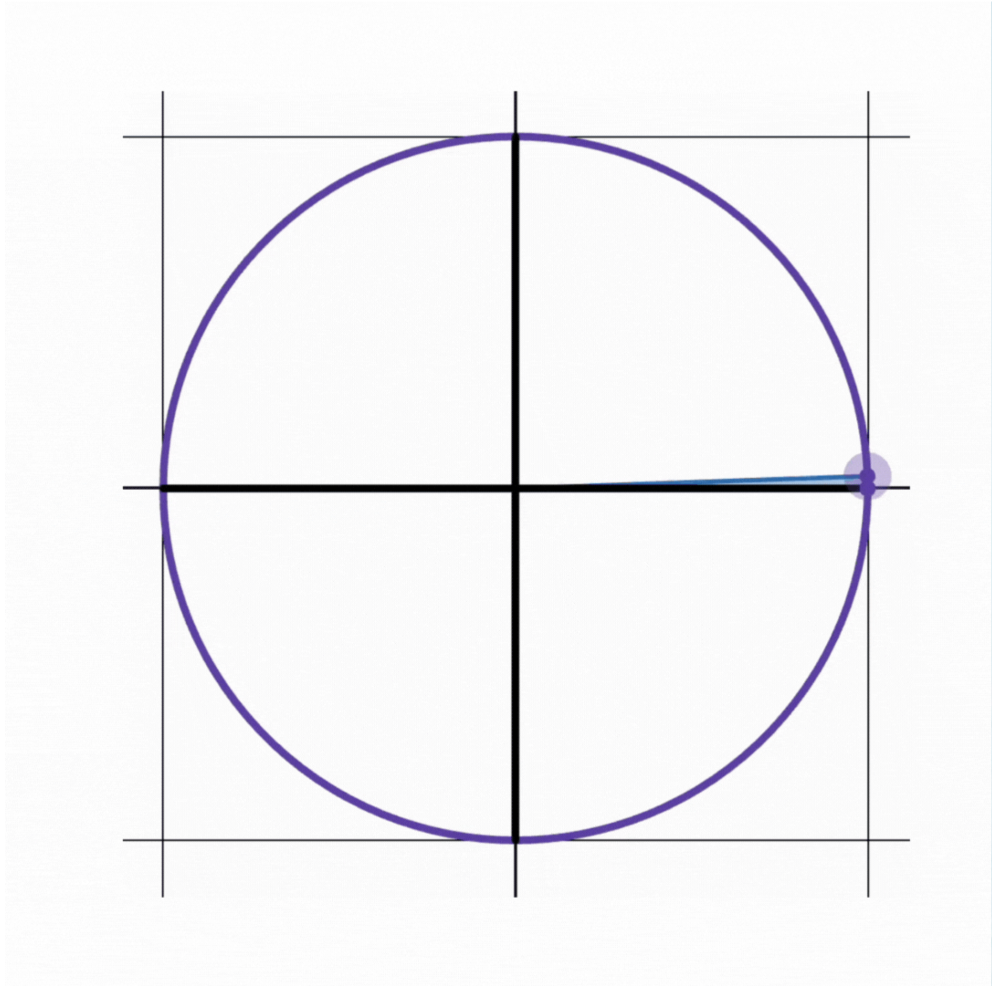


- **a.** Using exact values, determine the value of θ.
- **b.** Estimate the approximate area of the water wheel correct to the nearest square metre, by first determining the area of one of the eight segments of the wheel using the area of a triangle.
- **c.** The excess power (P) in watts generated by the water wheel over time (t) in hours from midnight can be modelled by $P = 120\cos(15t)$. Calculate the amplitude and period of the graph.
- **d.** Sketch the graph of $P = 120\cos(15t)$, for $0 \le t \le 48$, and determine at what times the power generated by the water wheel is equal to the power used by the manufacturing plant.
- **e.** When the graph is positive the water wheel generates more power than required. Determine the information that is represented when the graph is negative and the possible implications for the power plant in this scenario.
- **f.** Identify one benefit of using a water wheel as a means of power production.

Not all tasks are created equal



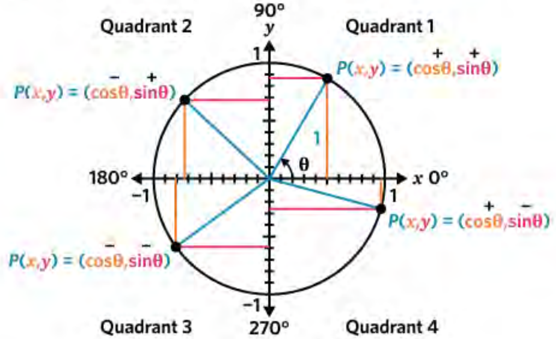
The unit circle



The unit circle

Key ideas

1. A unit circle is a circle with radius of 1 and centre (0,0) where every point $P(x,y)$ on the circle can be described in terms of the angle θ such that: $x = \cos\theta$ and $y = \sin\theta$, where $-1 \leq \sin\theta \leq 1$ and $-1 \leq \cos\theta \leq 1$. Positive angles, θ, are measured anticlockwise from 0°. Negative angles, θ, are measured clockwise from 0°.



2. When determining values using the unit circle, supplementary and complementary angle properties can be used to determine a reference angle that relates the angle, θ, to cosθ and sinθ in the first quadrant. The reference angle is always acute and can be used to approximate positive or negative cosθ and sinθ depending on the quadrant θ is located. Symmetry and angle properties within the unit circle can be used to determine the reference angle.

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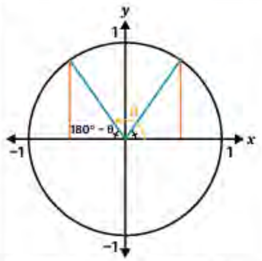
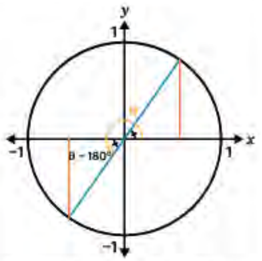
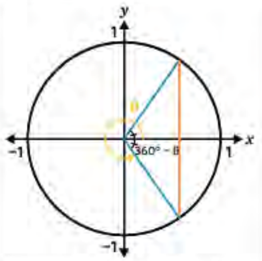
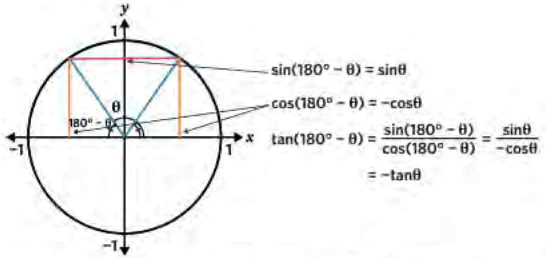
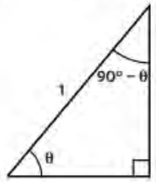
Quadrant 2	Quadrant 3	Quadrant 4
		
Reference angle = 180° − θ	Reference angle = θ − 180°	Reference angle = 360° − θ

Supplementary angles



Complementary angles add to 90°, so θ and 90° − θ are complementary.



For a right-angled triangle using the non-right angles, where $0° \leq \theta \leq 90°$

$\sin\theta = \cos(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$

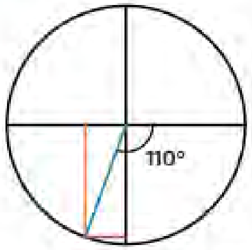
The unit circle

Spot the mistake

Select whether Student A or Student B is incorrect for questions 9 and 10.

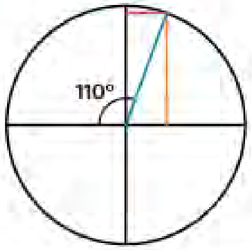
9. State the quadrant of the unit circle in which the angle $-110°$ is found.

A. Student A



$-110°$ is found in quadrant 3

B. Student B



$-110°$ is found in quadrant 1

10. If $\sin 120° \approx 0.86$ and $\cos 120° = -0.5$, determine the approximation of $\tan 120°$.

A. Student A

$$\tan 120° \approx \frac{0.86}{-0.5} \approx -1.72$$

B. Student B

$$\tan 120° \approx \frac{-0.5}{0.86} \approx -0.58$$

The unit circle

Problem solving

Question working paths

Mild 11, 12, 13	Medium 12, 13, 14	Spicy 13, 14, 15

11. Students are designing a trebuchet, as shown. The pivot point is 0.8 m above the ground and the arm is 1 m long from the pivot point to the basket. The arm does not pass 90° and the payload is released from the basket when the arm makes an angle of 80° with the horizontal. Determine the approximate height of the basket above the ground in metres when it releases the payload. Give your answer to two decimal places.

12. The length of a seesaw bench from the middle pivot point is exactly 1 m on both sides. To ensure safety, it is designed so that when one side hits the ground, the other side can rise to a maximum height at an angle of 40° from the ground. Estimate the maximum height of the pivot point in metres to one decimal place.

The unit circle

Reasoning

Question working paths

Mild 16 (a,b,c,e)	Medium 16 (a,b,c,e), 17 (a,b)	Spicy All

16. Maxum is practising a yo-yo trick where the yo-yo completes a full circle while the 1 m string is fully extended, skimming the ground, then returns to his hand. The rotation of the yo-yo on the 1 m long string can be modelled using a unit circle.

- **a.** Draw the unit circle that represents the movement of the yo-yo as it completes a full circle.
- **b.** Approximate the horizontal distance between the yo-yo and Maxum's hand when the string forms a 40° angle, to one decimal place.
- **c.** Approximate the horizontal distance between the yo-yo and Maxum's hand when the string forms a 230° angle, to one decimal place.
- **d.** What possible angles, θ, would be made if the yo-yo was 0.5 m from the ground?
- **e.** List some benefits in learning to use a yo-yo.

17. Complete the following questions using a unit circle.

- **a.** Determine sine, cosine, and tangent for 90°.
- **b.** Determine sine, cosine, and tangent for 180°.
- **c.** Using the answers from parts **a** and **b**, explain why tan90° and tan270° are undefined.

Algorithmic thinking

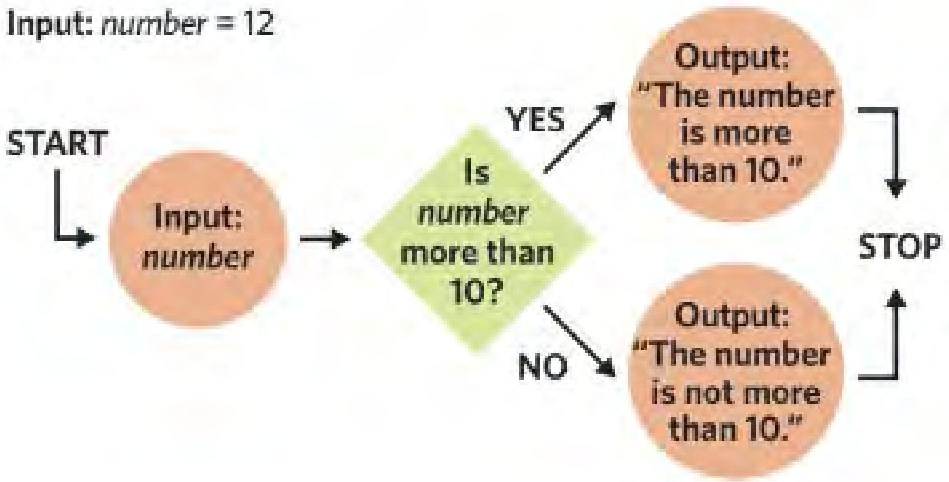
Year level	What's covered
Year 7	Flow charts and basic spreadsheets
Year 8	Loops and introduction to Desmos
Year 9	Introduction to pseudocode
Year 10	Data structures

key elements of algorithm design, including sequencing, decision-making and repetition, and representations of the ordered steps for an algorithm including through the use of pseudocode

the role of developing algorithms and expressing these through pseudocode to help determine and understand mathematical ideas and results

Algorithmic thinking

- Year 7



- Year 12

Question 13

The following algorithm applies Newton's method using a **For** loop with 3 iterations.

```
Inputs: f(x), a function of x
        df(x), the derivative of f(x)
        x0, an initial estimate

Define newton(f(x), df(x), x0)
    For i from 1 to 3
        If df(x0) = 0 Then
            Return "Error: Division by zero"
        Else
            x0 ← x0 − f(x0) ÷ df(x0)
    EndFor
    Return x0
```

The **Return** value of the function newton($x^3 + 3x - 3$, $3x^2 + 3$, 1) is closest to

A. 0.83333
B. 0.81785
C. 0.81773
D. 1
E. 3

Algorithmic thinking

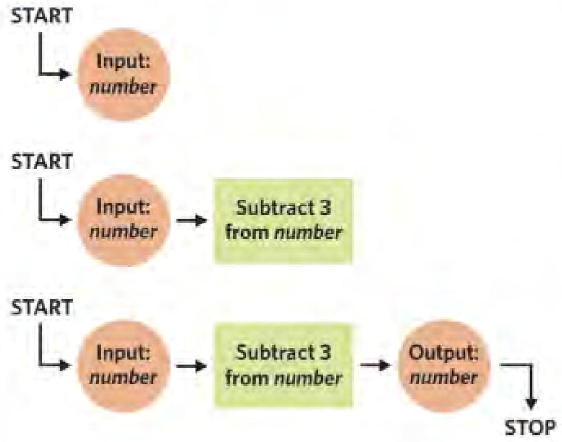
Worked example 1

Constructing flowcharts to represent algorithms

Construct a flowchart to represent each algorithm.

a. Take a number, subtract 3 from it, and output the result. WE1a

Working



Thinking

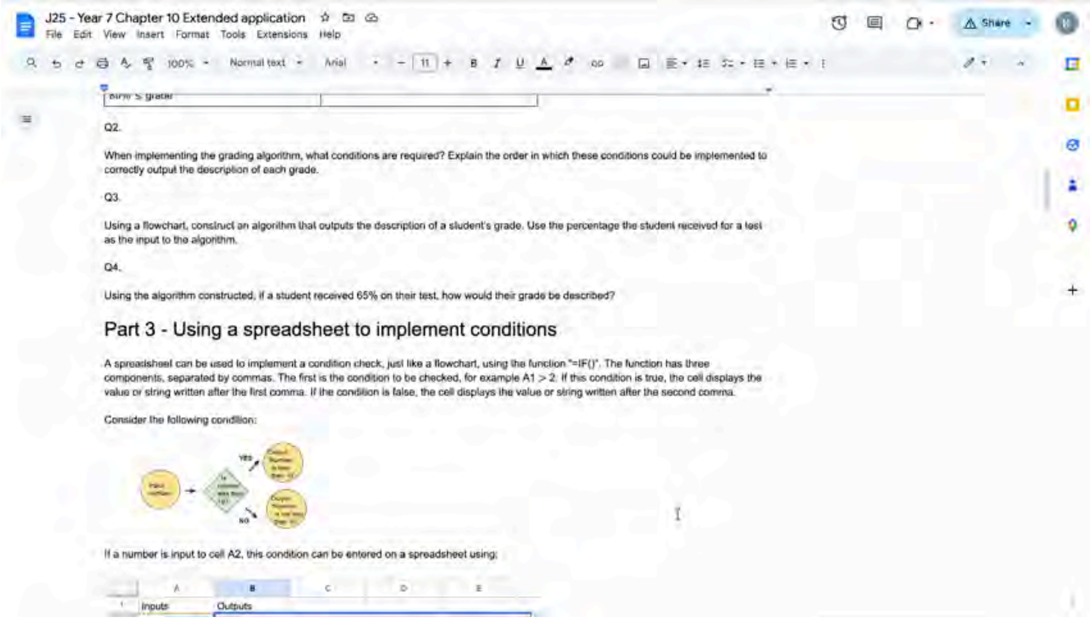
Step 1: Add a start and an input to create the variable *number*.

Step 2: Add an action to subtract 3 from it.

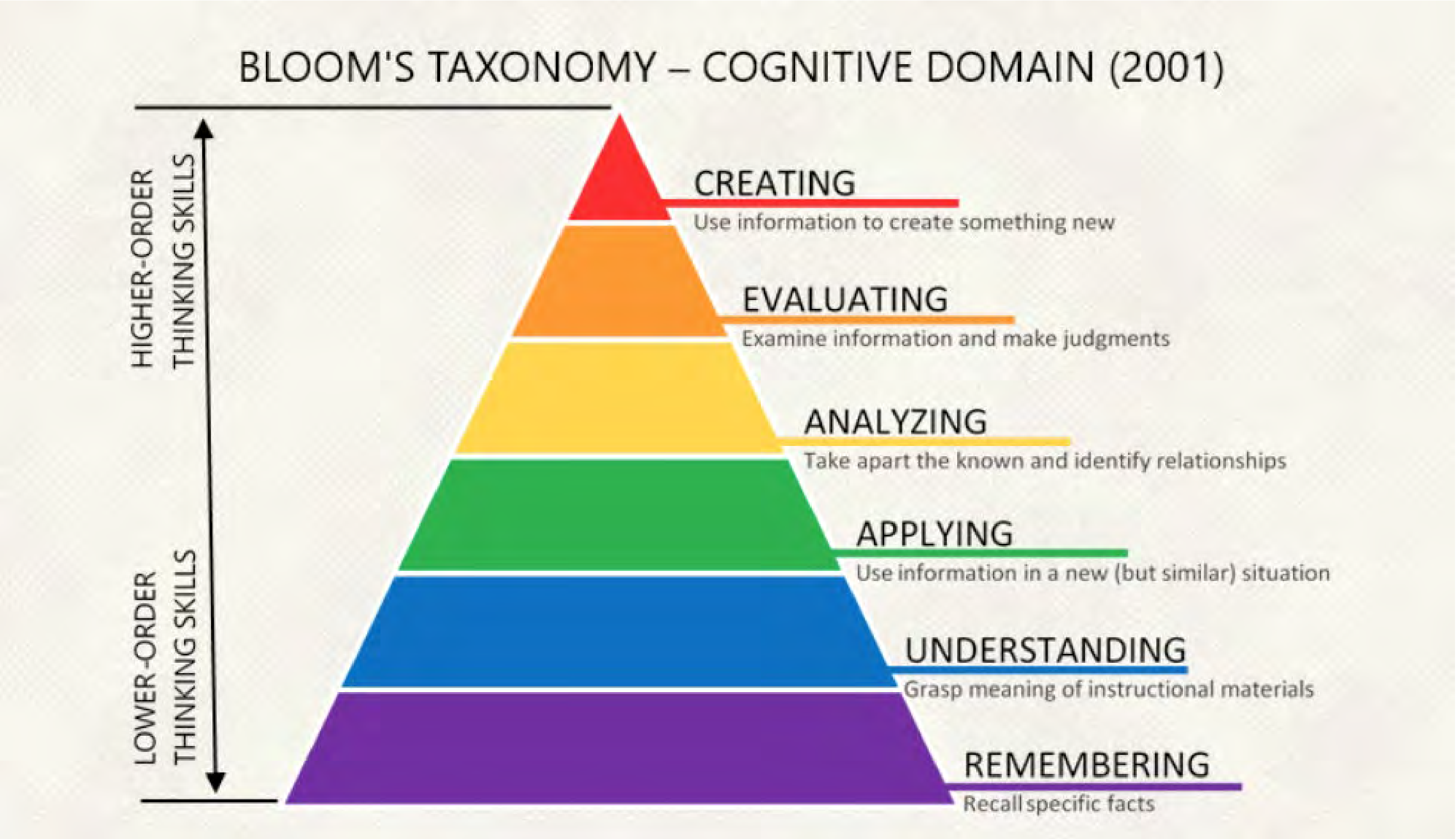
Step 3: Add an output with the variable *number* and add a stop.

Algorithmic thinking

- Application task



Investigation tasks + explicit teaching



Learning Design

Teachers are learning designers.
Maths lends itself to questioning.
Questioning leads to discovery.
Discovery leads to learning.

Thank you!

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